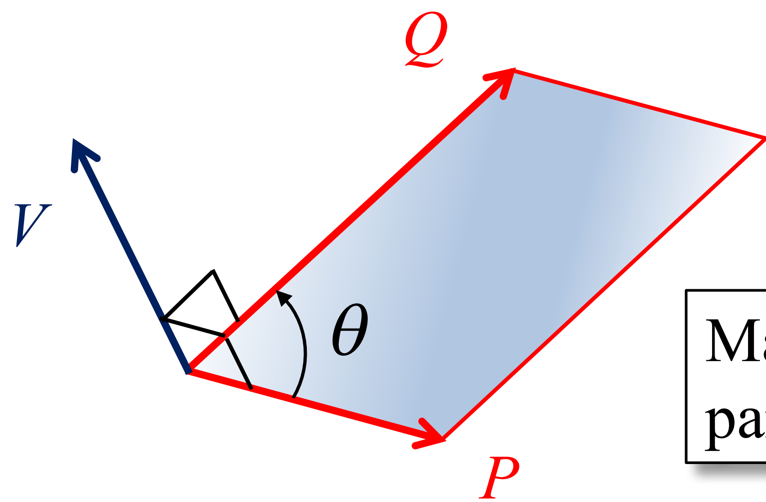
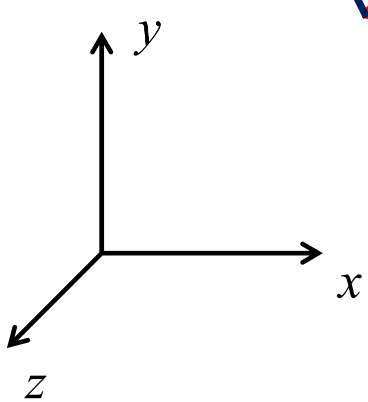
Vector Product

Steven Vukazich
San Jose State University

Definition of the Vector Product of Two Vectors

Two Vectors in space define a plane





$$\boldsymbol{V} = \boldsymbol{P} \times \boldsymbol{Q}$$

$$V = PQ \sin\theta$$

Magnitude of ***V*** is the area of the parallelogram defined by ***P*** and ***Q***

Direction of ***V*** is perpendicular to the plane defined by ***P*** and ***Q***

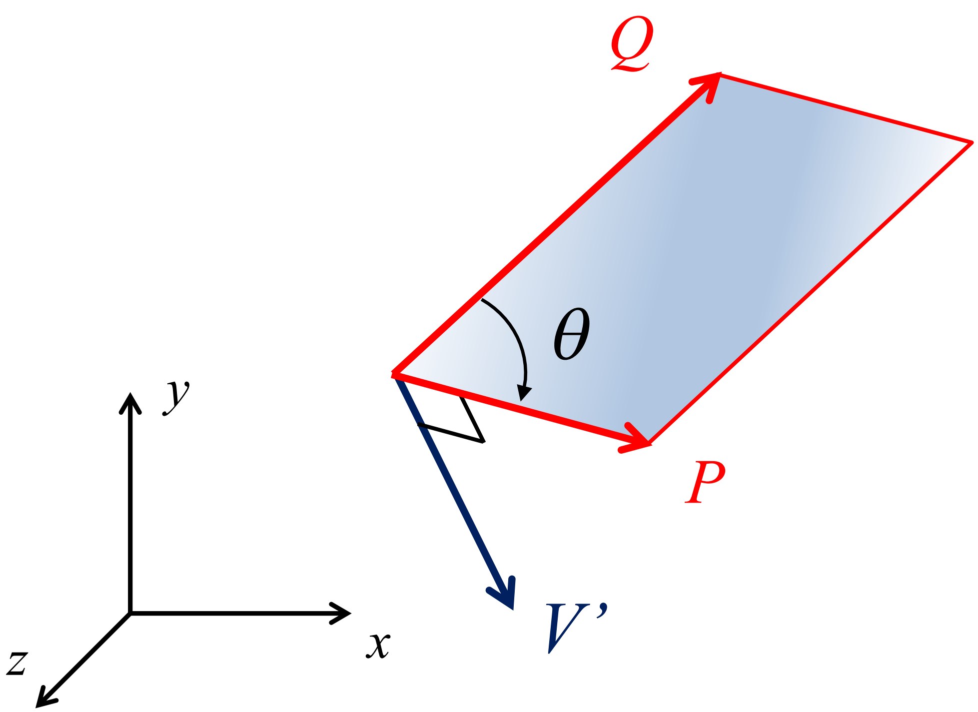
Sense of ***V*** is defined by the right-hand rule

Definition of the Vector Product of Two Vectors

Note that the order of the vector product operation changes the sense of the vector product

$$\boldsymbol{V'} = \boldsymbol{Q} \times \boldsymbol{P}$$

$$V' = PQ \sin \theta$$



Vector Products of Unit Vectors

$$\hat{\imath}\times\hat{\jmath} = \hat{k}$$

$$\hat{\jmath}\times\hat{\imath} = -\hat{k}$$

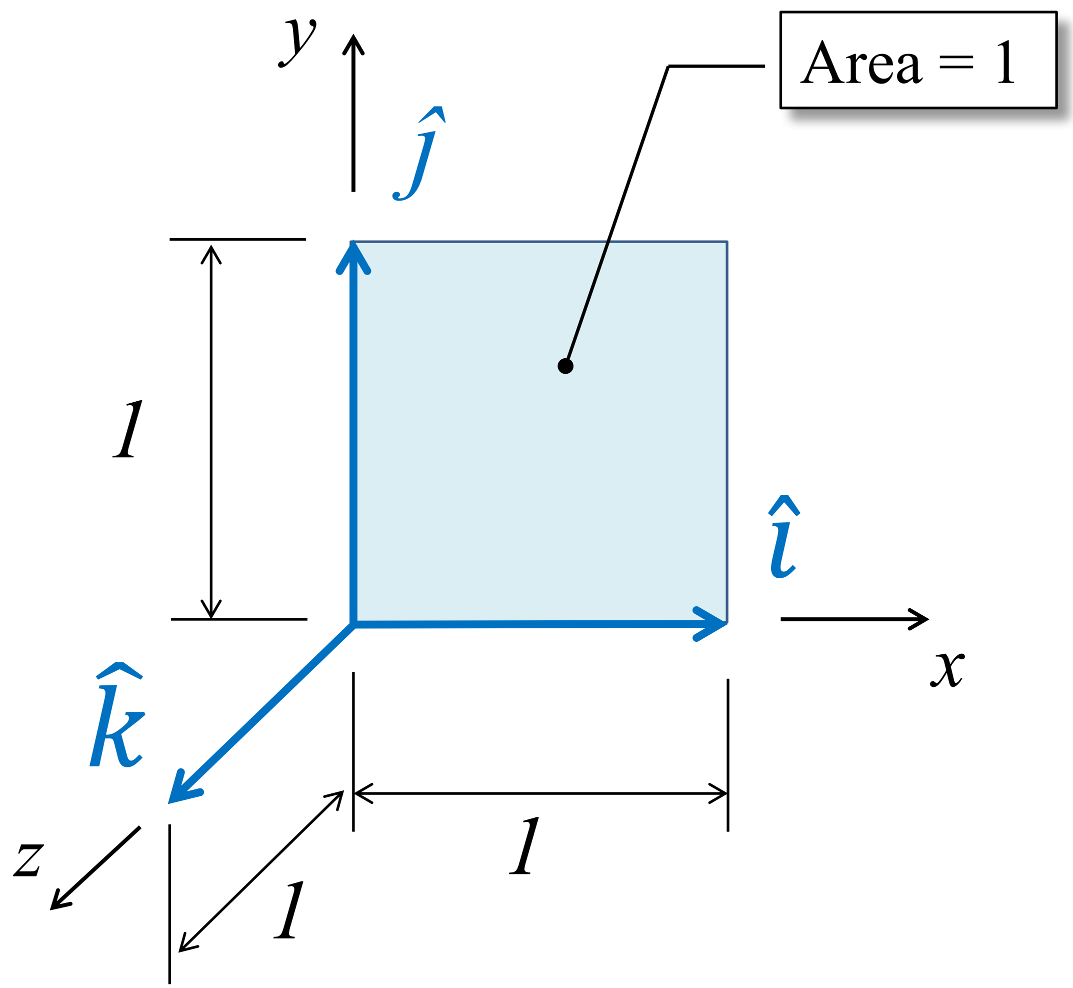
$$\hat{k}\times\hat{\imath} = \hat{\jmath}$$

$$\hat{\imath}\times\hat{k} = -\hat{\jmath}$$

$$\hat{\imath}\times\hat{\imath} = 0$$

$$\hat{k}\times\hat{k} = 0$$

Etc.



Vector Product of Two Vectors in Cartesian Vector Form

$$\boldsymbol{V} = \boldsymbol{P} \times \boldsymbol{Q}$$

P and ***Q*** expressed in Cartesian Vector Form

$$\boldsymbol{P} = P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k}$$

$$\boldsymbol{Q} = Q_x\hat{\imath} + Q_y\hat{\jmath} + Q_z\hat{k}$$

$$\boldsymbol{V} = \left(P_x\hat{\imath} + P_y\hat{\jmath} + P_z\hat{k}\right) \times \left(Q_x\hat{\imath} + Q_y\hat{\jmath} + Q_z\hat{k}\right)$$

$$\boldsymbol{V} = \left(P_yQ_z - P_zQ_y\right)\hat{\imath} + (P_zQ_x - P_xQ_z)\hat{\jmath} + \left(P_xQ_y - P_yQ_x\right)\hat{k}$$

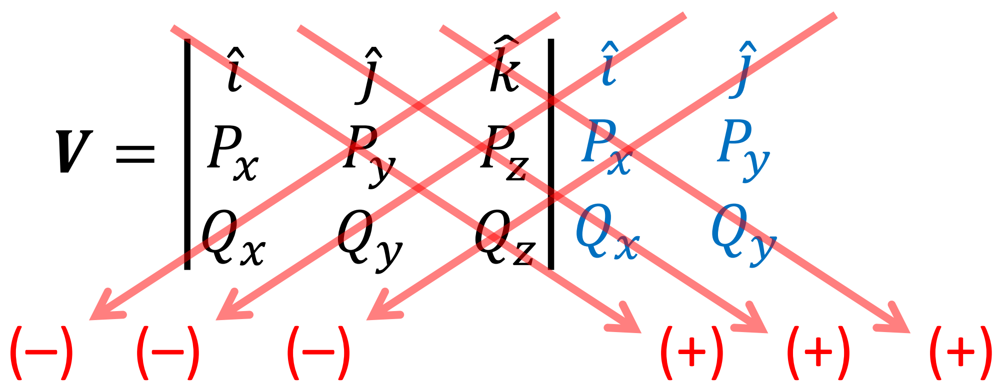
Vector Product of Two Vectors in Cartesian Vector Form

$$\boldsymbol{V} = \boldsymbol{P} \times \boldsymbol{Q}$$

Convenient "trick" to find vector product of two vectors in Cartesian Vector Form is to arrange the unit vectors and components in matrix form

$$\boldsymbol{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

Vector product is the determinate of the 3x3 matrix



$$\boldsymbol{V} = \left(P_y Q_z - P_z Q_y\right)\hat{\imath} + \left(P_z Q_x - P_x Q_z\right)\hat{\jmath} + \left(P_x Q_y - P_y Q_x\right)\hat{k}$$